

THE STEREOMETRICON.

ORIGINATOR: C. BAILLARGÉ, M.S.

MEMBER OF THE SOCIETY FOR THE GENERALIZATION OF EDUCATION IN FRANCE, AND
OF SEVERAL LEARNED AND SCIENTIFIC SOCIETIES; CHEVALIER OF THE
ORDER OF ST. SAUVEUR DE MONTE-REALE, ITALY; ETC., ETC.

MEASUREMENT OF ALL SOLIDS BY ONE AND THE SAME RULE.

UNIVERSAL APPLICATION OF THE PRISMOIDAL FORMULA.

THIRTEEN MEDALS AND SEVENTEEN DIPLOMAS AND LETTERS AWARDED THE AUTHOR
FROM RUSSIA, FRANCE, ITALY, BELGIUM, JAPAN, ETC.

PROMOTER: THOMAS WHITTY,
PROFESSOR AT ST. DENIS ACADEMY, MONTREAL.

Comprises 200 Solids representative of all conceivable elementary forms, as of
the Component parts of Compound bodies.

Name and description of each solid. What it is representative or suggestive of, or
that of which it forms a Component part.

Nature and name of opposite bases and of middle section as of lateral faces and
remainder of bounding Area, including every species of Plane,
Spherical, Spheroidal, and Conoidal figures.

Division I, classes I to X: plane faced Solids and Solids of single curvature.
Division II, classes XI to XX: Solids of double curvature.

Montreal:

PRINTED BY JOHN LOVELL & SON.

1880.

P5012
1881
B157

The EDITH and LORNE PIERCE
COLLECTION of CANADIANA



Queen's University at Kingston

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Member of the Society for the Generalization of Education in France and of several learned and scientific Societies; Chevalier of the Order of St. Sauveur de Mont-Réal, Italy; etc., etc., etc.

Measurement of all solids by one and the same rule.

Universal application of the prismoidal formula.

Thirteen Medals and seventeen Diplomas and letters awarded the author, from France, Russia, Italy, Belgium, Japan, etc.

PROMOTER: THOMAS WHITTY, professor at St. Denis Academy, Montreal, etc.

RULE: To the sum of the opposite and parallel end areas add four times the area of a section midway between and parallel to the opposite bases; multiply the whole by $\frac{1}{6}$ part of the length or height or diameter of the solid, perpendicular to the bases; the result will be the solidity or volume, the capacity or contents of the body, figure or vessel under consideration.

For application of the rule and examples of all kinds fully worked out, see "Key to Stereometricon."

For areas of all kinds, plane, and of single and double curvature, see also "Key to Stereometricon," with tables of areas of circles to eighths, tenths and twelfths of an inch, or any other unit of measure, tables of segments and zones of a circle, etc., etc., at end of "Key."

The tableau comprises 200 models, disposed in 10 horizontal and 20 vertical rows, series, families or classes. The solids may be indifferently placed, and numbered from the right or left and from below upwards.

The solids are representative of all conceivable elementary forms and figures, as of the component parts of all compound bodies.

DIVISION I.

Plane faced solids and solids of single curvature, or of which the surfaces are capable of being developed in a plane.

CLASS I.

Prisms.

Name of solid, objects of which it is representative or suggestive, or of which it forms a component part.

Reference to "Key to Stereometricon," for computation of contents and of factors necessary thereto.

Nature and name of opposite bases and middle section, of lateral faces and remainder of bounding surface.

Reference to page or paragraph of "Key" for calculation of areas and of factors necessary thereto.

NOTE.—The author uses the term "trapezium" and not "trapezoid," as the termination "oid" conveys the idea of a solid as paraboloid, hyperboloid, conoid, prismoid, etc.

For the same reason he uses the French "trapeziform" instead of trapezoidal.

**1.—The cube or hexaedron—
one of the five platonic bodies.**

Representative of a building or block of buildings or of one of the component parts thereof; a brick or cut stone, a pedestal, a die or dado; a pier or quay; a box, chest, package of merchandise or parcel; a cistern, bin, vat or other vessel of capacity; a pile of bricks, stones, lumber, books, etc., etc., etc.

"Key to Ster.," page 61, par. (78).

2.—A right isosceles triangular prism.

On end, a triangular block or building; on its base, a ridge roof; on one of its sides, the roof of a pent house or lean-to. "Key to Ster.," page 61.

3.—A right regular pentagonal prism.

On end, the base or component part of the shaft of an octagonal pier or column; on one of its sides, a baker's, butcher's or other van; an ambulance, etc.

"Key to Ster.," page 61.

4.—A right regular octagonal prism.

Base or shaft of a column, a pier or post, a bead, baluster, hand-rail, etc.

"Key to Ster.," page 61.

5.—An oblique hexagonal prism.

An inclined post or strut or the section of a stair-rail, a baluster on a rake, etc. Mitred section of a rail or bead.

See "Key to Ster.," page 64.

6.—Oblique rectangular prism.

On end, an inclined strut or post, etc.; on its parallelogram base, the pier of a skew bridge, portion of a mitred fillet, etc. See "Key to Ster.," page 64.

**7.—Oblique prism or parallelopi-
ped.**

Section of mitred fillet on an inclined or oblique surface, etc.

**8.—A right rectangular trapezi-
form prism, or a prism of which
the base or section is a rectan-
gular trapezium.**

On end, a pier or block of that shape; on its larger parallel face or base, the

Each of its three pairs of opposite and parallel faces or of its six faces or bases and middle sections, **perfect and equal squares**. For developed surface, see "Key to Ster.," page 131.

Representative of the floor, ceiling, walls or partitions of a rectangular room or apartment, or of the bases and sides of the various objects mentioned under the name of the solid. See "Key to Ster.," page 60.

Its opposite and parallel bases and middle section, **equal right-angled isosceles triangles**. Its sides or lateral faces, **rectangles**. For areas, see "Key to Ster.," pages 19, 22 and 60. Sides suggestive of those of objects alluded to.

Its opposite and parallel bases and middle section, **regular and equal pentagons**; sides or lateral faces, **rectangles**.

Areas suggestive of those of objects mentioned in adjoining column. "Key," pages 35 and 19.

Its parallel and opposite bases and section, **regular and equal octagons** its sides or lateral faces, **rectangles**. "Key," pages 35, 19.

Its parallel bases and section, **symmetrical and equal hexagons**; its sides, **parallelograms**. "Key to Ster.," pages 26, 19 and 63. Compute half of sym. hex. as a trapezium.

Two of its three pairs of opposite and parallel faces or bases and sections, **equal rectangles**; the other bases and section, **equal parallelograms**. "Key to Ster.," page 63.

Each of its three pairs of parallel faces or bases and sections, **equal parallelograms**.

Its opposite and parallel bases and section: on end, **equal rectangular trapeziums**; its lateral faces, **rectangles**; on either of its parallel sides or faces: its bases, **rectangles**; its lateral faces, **rectangles and trapeziums**

partially flat roof of a pent-house or lean-to; the base of a rectangular stack of chimneys on a sloped roof or gable, a corbel, etc. See "Key to Ster.," page 61.

9.—A right trapeziform prism.

On end, the splayed opening of a door or window or loop-hole in a wall; with broader base, a partially flat roof; on its lesser parallel base, a bin or trough or other vessel or vehicle of capacity, section of a ditch excavation or of a railroad embankment on level ground, a scow or pontoon.

10.—A right or oblique polygonal compound prism, decomposable into right or oblique triangular prisms or frusta of prisms.

An excavation or filling, etc.

A spoil bank or a borrowing pit.

See "Key to Ster.," pages 60 and 29.

May be treated indifferently as a prism or prismoid.

On end, its bases and section, **trapeziums**, and sides, **rectangles**; on either of its parallel faces, its bases and section, **rectangles**; its sides, **rectangles** and **trapeziums**.

N.B.—Its solid contents, like those of Nos. 2 and 8, may be computed either as prisms or prismoids.

Rule for solid content: multiply one-third the sum of the three vertical edges or depths of each of the component triangular prisms, or frusta of triangular prisms by the area of a section perpendicular to sides or horizontal, and add the results.

Page 67, rule II., "Key to Ster."

CLASS II.

Prisms, Frusta and Ungulae of Prisms.

11.—A right regular triangular prism.

On end, a triangular building, pier or lock; on one of its sides, the gable of a wall, the roof of a gabled house, etc.

"Key to Stereometicon," page 61.

Its parallel bases and section, equal **equilateral triangles**; its faces, **rectangles**. Compute as prismoid with rectangular bases, the upper base then being an **arris** or **line**.

12.—Lateral wedge or ungula of a right hexagonal prism, by a plane through edge of base.

Portion of a mitred bead or hand-rail, end of stair baluster under hand-rail, ridge roof of an octagonal tower against a wall; base of a chimney stack on a sloped roof or gable.

One of its parallel bases a **regular hexagon**; its middle base a **half hexagon** or **trapezium**; its upper base a **line**; its lateral faces a **line**, a **rectangle**, **triangles** and **trapeziums**; its sloped face a **symmetrical hexagon** or **2 trapeziums**, base to base

13.—Lateral ungula of a right hexagonal prism, by a plane through opposite angles of the solid.

Base of a chimney stack, vase or ornament on a sloped roof or gable, etc.

N.B.—This solid and the last, are not prismoids according to the definition thereof, page 163, par. (206), "Key to Ster.;" but the upper half, folded over

One of its opposite and parallel bases, a **regular hexagon**; the other, a **point**; its middle section a **half hexagon** or **two rectangular trapeziums** base to base; its lateral faces, **trapeziums** and **triangles**; its plane of section, a **symmetrical hexagon**, which, for area, regard as two equal **trapeziums** base to base, compute and add.

See "Key to Ster.," page 29.

and applied to the lower half, evidently completes the prism, and hence the solidity is exactly obtained by the prismoidal formula, as it is of a like frustum of a cylinder or of an ungula thereof by a plane through edge of base.

Or the symmetrical hexagon may be decomposed into a rectangle and two equal triangles, for computation of area

14.—Central wedge or ungula of a right hexagonal prism; a prismoid.

A wedge, the ridge roof of a tower, the base of a chimney stack, vase or ornament between two gables.

One of its parallel bases, a hexagon; the other, a line; its middle section, a symmetrical hexagon or two trapeziums, base to base; its lateral faces, triangles and trapeziums.

See "Key to Ster.," page 29.

15.—An oblique trapeziform prism.

The partially flat roof to a dormer window, the roof of a building abutting against another roof, the splayed opening of a basement window, a mitred portion of a batten or moulding, section of a ditch excavation, or of an embankment on a slope.

Treated as a prismoid: its opposite and parallel bases, unequal rectangles; its lateral faces, trapeziums.

The factors of its middle section arithmetical means between those of its opposite and parallel bases.

16.—An oblique triangular prism.

The roof of a dormer window or of a wing to a house with a sloped roof, a mitred moulding or fillet, etc.

Treated as a prismoid: one of its opposite and parallel bases, a rectangle; the other, a line; its lateral faces, equal triangles and parallelograms.

17.—Frustum of a right triangular prism.

Ridge roof of a building against a wall, a mitred moulding, etc.

As a prismoid: one of its parallel bases, a rectangle; its opposite base, a line; its middle section, a rectangle.

18.—Irregular frustum of an oblique triangular prism.

Ridge roof of a building of irregular plan abutting on the unequally sloped roof of another building, etc.

Considered as a prismoid: one base, a trapezium, the other, a line; its middle section, a trapezium; its ends, non-parallel triangles; its sides, trapeziums.

19.—A right prism on a mixtilinear base.

On end, the unsplayed opening of a door or window in a wall, etc.

Note, for area of segment of circle or ellipse, see "Key to Ster.," pages 38, 44, 51, 53, 57, tables II., III., IV., VII.

Parallel bases and section mixtilinear figures, decomposable into a rectangle and the segment or half of a circle or ellipsis; the lateral face, a continuous rectangle.

Note.—The segment of a circle or ellipse may be equal to, less or greater than a semi-circle.

20.—Regular frustum of an oblique triangular prism.

A ridge roof, mitred fillet, etc.

As a prismoid: one base, a rectangle; the other, a line; the middle section, a rectangle.

Frusta of Prisms, Prismoids, Wedges.

21.—The dodecahedron, or twelve-sided solid, one of the five platonic bodies.

Assemblage of twelve equal pyramids with pentagonal bases, their apices or summits meeting in the centre of the solid or of the circumscribed sphere.

The capital or intermediate section of a pentagonal shaft or column, a finial or other ornament.

The six pairs of parallel bases or twelve component faces of the solid, **equal and regular pentagons**; the middle section a **regular decagon**, the side of which is equal to half the diagonal of the pentagon, for area of which see "Key to Ster.," page 36, rule II; or compute one of the component pyramids and multiply by twelve. For developed surface, see "Key to Ster.," page 132.

22.—A rectangular wedge, the head or heel broader than the blade or edge.

The frustum of a triangular prism, or may be treated as a prismoid, using either of its three pairs of parallel bases.

An inclined plane, a low pent roof, an ordinary wedge, etc.

On end: its opposite and parallel bases, a **rectangle** and a **line**; its middle base or section, a **rectangle**. On one of either of its other two pairs of parallel bases: one base, a **trapezium**, the other, a **line**; the middle section a **trapezium**; side faces, a **rectangle** and **triangles**.

23.—A rectangular wedge or inclined plane, the head or heel of equal breadth with the edge or blade.

A right triangular prism. Body of a dormer window or base of a chimney stack on a low or steep roof, etc.

Each of its three pairs of parallel bases, a **rectangle** and a **line**; its middle sections, **rectangles**, respectively equal to half the corresponding base. May also be treated as a triangular prism, with bases and section **equal triangles**.

24.—An isosceles wedge, the edge or blade broader than the heel.

May also be considered, the frustum of a triangular prism or a prismoid with three pairs of parallel bases.

As a prismoid: one of its pairs of parallel bases, a **rectangle** and a **line**; middle section, a **rectangle**; each other pair of parallel bases, a **trapezium** and a **line**; middle section, a **trapezium**.

25.—Frustum of a right rectangular trapeziform prism, or a prismoid.

A roof, partially flat, abutting against vertical wall at one end and in rear, against a sloped roof at the other, etc.

As a prismoid: its opposite and parallel bases, **rectangles**; the longer side of the one corresponding to the shorter side of the other; its middle section, a **rectangle**; all its lateral faces, **trapeziums**.

26.—Irregular frustum of an oblique trapeziform prism.

A roof between two others, not parallel, irregular section of a ditch or embankment.

As a prismoid: its opposite and parallel bases and middle section, **trapeziums**; its lateral faces, **trapeziums**.

Factors of middle section arithmetic means between those of the bases.

27.—Frustum of a right isosceles trapeziform prism, a prismoid.

On its larger base, a roof, section of an embankment, etc.; on its lesser base, a bin or vessel of capacity; the capital of a pilaster, a corbel; on end, a splayed opening in a wall.

28.—Frustum of an isosceles triangular prism, a prismoid.

Ridge roof with ends unequally sloped, mitred moulding, etc.

29.—Frustum of a trapeziform prism, a prismoid.

A flat roof, etc.; on its lesser parallel base, a bin or reservoir, a vehicle of capacity, a scow, a pontoon; on end or its parallel faces vertical, the splayed opening of a window.

30.—A prismoid on a mixtilinear base.

The roof of a building, circular at one end or coved ceiling of a room; on its lesser base, a bathing tub, etc.; vertically, the splayed opening of a circular headed window in a wall.

As a prismoid: its opposite and parallel bases and middle section, **rectangles**; lateral faces, **trapeziums**.

In all such solids, the half way factors need never be measured, as they are always means between the parallel bases of the trapezium faces.

As a prismoid: one of its opposite and parallel bases, a **rectangle**; the other, a **line**; its middle section, a **rectangle**. "Key to Ster.," page 19.

As a prismoid: its opposite parallel bases and middle section, **rectangles**; its lateral faces, **trapeziums**. Factors of intermediate section or middle base, arithmetic means between those of the end bases.

"Key to Ster.," page 29.

Its opposite and parallel bases and middle section, **mixtilinear figures**; the one a **rectangle** and a **semi-circle**; the other two, **rectangles and semi-ellipses**; its arched end developed, a sort of trapezium with curved bases; its area equal to half sum of bases by mean breadth or height.

CLASS IV.

Prismoids, etc.

31.—The icosahedron, or twenty-sided solid; one of the five platonic bodies.

An assemblage of twenty equal pyramids on triangular bases, their apices or summits meeting in a common point, the centre of the solid or of the circumscribed or inscribed sphere.

A finial or other ornament, etc. More expeditious to treat it for solidity by computing one of its component pyramids, and multiplying the result by twenty.

The ten pairs of parallel bases or twenty component faces of the solid are equal **equilateral triangles**. Its middle section, a **regular dodecagon**. Its middle section parallel to two opposite apices or to the bases of any two opposite pentagonal pyramids of the solid, a **regular decagon**, whose side is equal to half that of one of the edges of the solid. For developed surface, see "Key to Ster.," page 133.

32.—A prismoid, both its bases, lines. Irregular triangular pyramid.

Dormer or gablet abutting on a sloped roof. Component section of No. 79. See "Key to Ster.," page 165, par. (212).

Its opposite bases—considering the solid as a prismoid resting on one of its parallel edges—**lines**; its middle section a **rectangle**. See "Key to Ster.," page 164, par. (208).

33.—A prismoid on a trapeziform base.

A cutting or embankment, etc.

One of its parallel bases, a **trapezium**; the other, a **line**; its middle section, a **trapezium**.

34.—A railroad prismoid on a side slope.

Section of a railroad cutting or embankment on ground, sloping laterally or in one direction only.

Its end sections or bases and middle parallel section **equal quadrilaterals**. For area of which see "Key to Ster.," page 30.

This prismoid is a prism on an irregular base, and may be so treated.

35.—A railroad prismoid on a grade and side slope, or on ground sloping both laterally and longitudinally.

Its narrow base upwards, an embankment; the same downwards, a cutting or excavation.

Its opposite and parallel end bases and middle section, **quadrilaterals**, the factors of the middle section being all arithmetic means between those of the corresponding end areas.

36.—A square or rectangular prismoidal stick of timber.

A squared log, a tapering post, the shaft of a chimney or high tower, a reducer between rectangular conduits of unequal size, etc.

Note.—25 per cent. of the whole or true content is $33\frac{1}{3}$ per cent., or one-third of the erroneous result.

Its end bases and middle section, **squares or rectangles**.

Timber is usually measured by multiplying its middle section into its length. This gives an erroneous result; the more tapering the timber is, the more so. If it tapered to a point the error would be 25 per cent., or one-quarter of the whole in defect.

37.—A prismoidal stick of waney timber.

A log of waney timber; on end, the shaft of a chimney, a high tower, a tapering post.

Its opposite bases and middle section, **symmetrical octagons**, for area of which see "Key to Ster.," page 176, par. (272), or squares or rectangles with chamfered corners or angles.

38.—A concavo-convex prismoid or curved wedge.

A corbel, spandrel, finial, etc.; a brake, a cam, etc. "Key to Ster.," par. (141).

Its opposite bases, a **rectangle** and a **line**; its middle section, a **rectangle**; its developed faces, **trapeziums** sides, **mixtilinear triangles**.

39.—A recto-concave prismoid, or frustum of a curved wedge.

A corbel, spandrel, buttress, etc. May be decomposed, as also No. 38, into two sections for more exact computation of solid contents.

Its opposite and parallel bases and middle section, **rectangles**; its developed faces, **trapeziums**; its lateral faces, **mixtilinear trapeziums**.

For areas see "Key to Ster.," page 57.

40.—Frustum of a rectangular trapeziform prism, a prismoid.

A flat roof in a rectangular corner; on its lesser base, an angular corbel, a sink, cistern, bin, etc.

As a prismoid, its opposite and parallel bases and middle section, **rectangles**; its lateral faces, **trapeziums**.

"Key to Ster.," page 104, par. (141).

Prismoids, etc.

41.—The octahedron or eight-sided figure ; one of the five platonic bodies.

Assemblage of eight equal pyramids on triangular bases, their apices meeting in a common point, the centre of the solid ; or two quadrangular pyramids, base to base.

Its four pairs of parallel bases or eight component faces, **equilateral triangles** ; its middle section, a **regular hexagon** ; its middle section through opposite apices and perpendicular to intervening arris or edge, a **lozenge** ; through four apices, a **square**. For developed surface see "Key to Ster.," page 132.

42.—A prismoid, one of its bases a square, the other an octagon.

Base or capital of a column, roof of a square tower, a tower, pier, vessel of capacity, component section of a steeple, etc.

Its opposite and parallel bases, a **square** and an **octagon** ; the middle section, a **symmetrical octagon** ; its lateral faces, **triangles** and **trapeziums**. For area of symmetrical octagon, see "Key to Ster.," par. (272).

43.—A prismoid, its opposite bases, a square and a circle.

Base or capital of a column, roof of a square tower, a tower, pier, vessel of capacity, a lighthouse, a section of a steeple or belfry, a reducer between a square and circular conduit.

One of its opposite and parallel bases a **square** ; the other, a **circle** ; the middle section, a **mixtilinear figure** or a **square with rounded corners**. Its lateral surface capable of development into a plane **trapeziform figure**, one base circular, the other polygonal.

44.—A prismoid, its bases unequal squares set diagonally.

Representative of the same objects as solids, Nos. 42 and 43.

Its opposite bases **unequal squares** set diagonally to each other ; the middle section, a **symmetrical octagon** ; its lateral faces, **triangles**.

45.—A prismoid, its bases a hexagon and a rectangle.

Representative of nearly the same objects as the three last solids.

One of its bases, a **hexagon** ; the other a **rectangle** ; its middle section a **symmetrical octagon** ; its lateral faces, **rectangles** and **triangles**.

46.—The lateral frustum of a rectangular prolate spindle.

Roof of a square tower, component part of a steeple, etc.

Its parallel bases and section, **squares** ; its lateral surface, **mixtilinear figures** capable of development into plane surfaces. For area of these see "Key to Ster.," page 57.

47.—A prismoid, its bases, an ellipse and a square.

A reducer between an elliptic and square conduit, a roof, etc.

Its middle section, a **mixtilinear figure** or **approximate oval**. Its lateral surface developed, a **curved trapezium**, one base curved, the other polygonal. See "Key to Ster.," p. 166.

48.—A prismoid, its bases a symmetrical hexagon and a line.

Ridge roof, coping or finial to a post, daniel ornament, etc.

Its middle base, a **symmetrical octagon**; its lateral surface, **triangles**. For symmetrical hexagon, area equal to double that of half the figure, which is a trapezium.

49.—A prismoid, its bases a symmetrical hexagon and a lozenge.

Flat roof, ornament, etc.; on its lesser base, a fancy basket, a disk, etc.

Its middle section or base, a **symmetrical decagon**; its lateral faces, **triangles**. Area of hexagon, double that of component trapezium.

50.—A groined ceiling or the half of a rectangular oblate spindle.

A roof, panel ornament, etc. For more exact computation of contents, decompose into two parts.

Its base and middle section, **squares**; its opposite base, a **point**; its lateral faces, **mixtilinear figures**.

For areas of mixtilinear figures see "Key to Ster.," page 57.

CLASS VI.

Pyramids and Frusta of Pyramids.

51.—The tetrahedron, or four-sided figure; one of the five platonic bodies. A regular triangular pyramid.

Apex roof of a triangular building, finial or other ornament, the component element of the icosahedron and octahedron.

Its base and middle section, **equilateral triangles**, the lesser equal in area to one-quarter the greater, its upper or opposite base, a **point**; its faces, **triangles**. For development of surface see "Key to Ster.," page 131. For area of bases and faces, see page 36, rule II.

52.—A regular square or rectangular pyramid.

The spire of a steeple, a pinnacle, roof of square tower, a bin, a vessel of capacity, a finial or other ornament, etc.

One of its parallel bases, a **square**; the other, a **point**; its middle section, a **square**, of which the area is one quarter that of the base. Lateral faces, **isosceles triangles**.

53.—A pyramid, two of its faces perpendicular to base. The ungula of a rectangular prism on either of its bases.

An apex roof, section of cutting or embankment, component portion of other solids, a roof saddle.

Its base and middle section, **triangles**; apex, a **point**. Factors of middle section half those of the base.

Affords a demonstration of the theorem that in right-angled spherical triangles the sines of the sides are as the sines of the angles.

54.—Frustum of a right triangular pyramid.

Roof, base or capital of a post or column, base of a table-lamp or vase, a vessel of capacity, component section of other solids.

Its parallel bases and middle section **similar triangles**; lateral faces, **trapeziums**. Factors of section arithmetic means between those of bases.

55.—Frustum of an oblique triangular pyramid.

That roof of triangular building abutting against a sloped or battered wall ; portion of a ditch excavation, component portion of other solids.

Its bases and middle parallel section, **similar triangles** ; lateral faces, **trapeziums** ; factors of section, arithmetic means between those of the bases. For areas see "Key to Ster.," pages 19, 22 and 29.

56.—Frustum of a right rectangular pyramid.

Flat roof to tower ; reducer between conduits of varied size, component portion of an obelisk, capital or base of a post or column, a bin, vat or other vessel of capacity, the body of a lantern, etc., etc.

Its opposite bases and middle section, **squares or rectangles** whose factors, or sides are each equal to half the sum of the corresponding sides of the bases, or arithmetic means between them. For areas see "Key to Ster.," pages 19 and 29.

57.—A regular octangular or octagonal pyramid.

Roof of a tower, spire of a steeple, final or other ornament, a funnel, strainer or filler, etc.

Its base and middle section, **similar octagons** ; lesser area one-quarter of the greater ; its upper base or opposite one, an apex or a **point** ; lateral faces, **isosceles triangles**.

58.—The frustum of a regular octagonal pyramid.

On its broader base, a roof, tower, pier, quay, component part of a steeple, etc. ; base of a column, lamp or vase, etc. ; on its lesser base, a vat, bin, vase, or other vessel of capacity ; the body of a lantern, etc., etc.

Its opposite and parallel bases and middle section, **regular octagons** ; factors of section means to those of the bases ; its lateral faces, **trapeziums**. For expeditious mode of arriving at area of octagon, see "Key to Ster.," page 176 or page 26, rule II. Developed surface a regular polygonal sector or trapezium.

59.—Irregular and oblique pyramid on a quadrilateral base.

Apex roof of an irregularly shaped building against a battered wall or roof, a roof saddle, etc.

Its base, a **quadrilateral or irregular trapezium** ; its summit or apex, a **point**. Middle section similar to base and equal in area to one-quarter that of base.

60.—Frustum of a pyramid with non parallel bases.

Decomposable into the frustum of a pyramid with parallel bases, and an irregular pyramid, by a plane parallel to the base and passing through the nearest corner or point of the upper, or non parallel base.

When decomposed for computation of solid contents : bases and section of frustum, **similar triangles** ; bases and section of component pyramid or upper portion, **similar quadrilaterals**. This pyramid has its base in one of the lateral faces of the proposed solid.

CLASS VII.

Cylinder, Frusta and Ungulae.

61.—A right cylinder or infinitary prism.

A tower or circular apartment ; a bin,

Its parallel bases and middle section, **equal circles** ; its lateral surface developed in a plane, a **rectangle** ; its

vat, tub, bucket, pail, vase, drinking vessel, cauldron or other vessel of capacity; a road or other roller; the cylinder of a steam or other engine; a gasometer, the barrel of a pump, etc., etc., etc.

62.—Frustum of lateral ungula or wedge of a right cylinder.

May represent a cylindrical window or opening in a sloped roof abutting to a vertical wall or surface, the liquid in a closed cylindrical vessel held obliquely, base to chimney or vase partly on a horizontal, partly on a gabled wall.

63.—A rectangular circular ring;

The difference between two concentric cylinders, or a solid annulus.

Horizontal section of a tower wall, cross section of a brick, iron or other conduit, section of a boiler, vat, tub, or other vessel of capacity, etc., etc.

64.—Central ungula or wedge of a right cylinder.

Ridge roof of a tower, a wedge, loop hole in a wall, component portion of compound solid, a finial or other ornament, a strainer, etc.

65.—Frustum of central wedge or ungula of cylinder No. 64.

Flat roof of tower or other building, base or capital of rectangular pillar, vessel of capacity, component portion of compound solid, base of chimney, stack or vase between two gables.

66.—Lateral ungula of right cylinder or recto-cylindrical wedge.

Lunette or arched headway of a door or window, etc., in a sloped roof, component of a compound solid, the liquid in an inclined cylindrical vessel, base of a salient chimney shaft over a roof, etc., etc.

height, that of the cylinder; its length, the circumference of the solid.

For areas of circles calculated to eighths, tenths and twelfths of unity, see tables II., III., IV. at end of "Key to Ster."

Its base, a circle; its opposite base, a semi-circle or other segment; its middle section, a segment greater than a semi-circle; its plane of section the segment of an ellipsis; its cylindrical surface decomposable by lines parallel to bases into trapeziums. For areas of segments, see table VIII., "Key to Ster.," pages 53, 38, 44.

Its bases and parallel section, concentric annuli; its interior and exterior surfaces, continuous rectangles. The area of annulus equal to the difference of the inner and outer circles, of to the breadth of annulus into half the sum of its circumferences. See "Key to Ster.," page 39.

Its base, a circle; its opposite base, a line; its middle section, the zone or a circle; its sloped faces, each a semi-ellipsis. Its cylindrical surface decomposable into trapeziums by arcs parallel to base. See tables II., III., IV. IX., of "Key to Ster.," also pages 38, 46, 53.

Its greater base, a circle; its lesser, base, the central zone of a circle; its intermediate base, the zone of a circle; its lateral faces, equal segment of equal ellipses. Its cylindrical surface decomposable into trapeziums parallel to bases. See "Key to Ster.," page 51.

Its base, a semi-circle; its intermediate base or middle section parallel to base, also a segment; its opposite base, a point; its plane of section or sloped face, a semi-ellipsis. Its curved surface developed an approximate parabola, trapeziums, etc. See "Key to Ster.," pages 38, 44, 51, tables II., III., IV., VIII.

67.—Frustum of lateral wedge or ungula of a right cylinder.

Lunette to arched opening in a sloped roof or ceiling, abutting on a vertical wall or surface; liquid in an inclined closed cylindrical vessel; base of engaged column against a battered wall, etc.

Its parallel bases and middle section, segments of a circle, less than more than, and equal to half; sloped face, the **excentric zone of an ellipsis**; cylindrical surface, trapezium parallel to base. For areas of segment, see "Key to Ster.," page 44, rule I., rule II., table VIII.; for zone of ellipsis, see page 53, art. (62).

68.—Irregular ungula or wedge of right cylinder.

Lunette to a partially circular opening in an inclined ceiling, etc. Component portion of a compound solid. For areas, see "Key to Ster.," pages 44, 46, 53, articles (61) and (62), tables VIII. and IX.

Its base, the segment of a circle greater than half; its opposite base, a line; its middle section, an **eccentric zone of a circle**; one of its side faces, the segment of an ellipsis; the other plane face, an **eccentric zone of an ellipsis**.

69.—Concave-convex prismoid or cylindric-cylindrical solid or concave frustum of a wedge or ungula of right cylinder.

Deposit of sediment in a cylindrical sewer, section of additional excavation or filling, or difference between two lunettes.

One of its bases, the lune of a circle greater than a semi-circle; the other the lune of a circle less than a semi-circle; the middle section, a lune equal or thereabouts to a semi-circle. Its side surfaces, convex and concave approximate trapeziums. For areas of lunes, see "Key to Ster.," page 47.

70.—Frustum of an oblique cylinder.

May be decomposed into an oblique cylinder and the ungula of one by a plane parallel to base, and passing through nearest point of other base.

When decomposed, its bases and section ellipses; the base of ungula, an ellipsis equal to each of those of the inclined cylinder; its middle section half an ellipsis. For ungulae, see Nos. 72, 73, 75.

CLASS VIII.

Oblique Cylinder, Frusta, Ungulae Cylindroids, etc.

71.—Oblique cylinder or infinitary prism.

Mitred section of conduit, hand rail, moulding; inclined column, post, strut or brace, etc.; inclined cylindrical opening in a wall, etc.

Its parallel bases and section, equal ellipses; its lateral surface capable of development into a plane mixtilineal figure. See "Key to Ster.," fig. n page 57. For area of ellipsis, see page 51 of same.

72.—Obtuse frustum or ungula of oblique cylinder.

Oblique lunette inclined upwards or arched headway to a circular or elliptical opening in a sloped roof or ceiling. Component mitred portion of hand-rail, bead molding, etc.

One of its opposite bases, an ellipsis of slight eccentricity; its opposite base, a point; its middle section, a semi-ellipsis equal to half of base; its plane of section or lateral face, an ellipsis of greater eccentricity; its lateral cylindrical face developed, a figure like m page 57 of "Key."

73.—Acute frustum or ungula of oblique cylinder.

Representative of same as No. 72, but inclined downwards.

Same as No. 72. For developed cylindrical surface, see fig. h, page 57 of "Key to Stereometicon."

For area of ellipsis, "Key to Ster.," pages 51 and 53.

74.—Concave ungula or frustum of oblique cylinder.

Representative of same as No. 73, but in arched roof or ceiling instead of sloped roof.

Same as No. 73, with curved instead of plane section. Its cylindrical surface developed similar to fig. h, page 57 of "Key;" its curved or concave section developed an oval or fig. like a, page 57, "Key to Ster."

75.—Frustum, ungula or wedge of right cylinder.

Base of chimney shaft on sloped roof, or same as No. 72 not inclined.

Same as No. 72. For developed cylindrical surface, see fig. g; for ellipsis, fig. b, page 57, "Key to Ster."

76.—A cylindroid; its bases, a circle and an ellipsis; infinitary prismoid.

Base or capital of elliptic column, reducer or connecting link between a circular and an elliptic conduit; a tub, vat or other vessel of capacity; a hat with elliptic or oval head and a circular crown, etc.

Its middle section, an ellipsis of which the conjugate or lesser diameter or axis is an arithmetic mean between those of the opposite bases. For area of circle, see tables II., III., IV., and of ellipses, page 51, "Key to Ster." Lateral surface developed, a **plane trapeziform fig**; its greater base, convex; lesser, concave; its area, equal periphery of middle section into mean height.

77.—Cylindroid or infinitary prismoid; its bases, an ellipsis and a circle.

Same as No. 76, or frustum of a conic metallic vessel, which has become flattened or battered at one end.

Its lateral surface develops into a **plane trapeziform figure**, with greater periphery convex; and lesser concave. Area equal to periphery of middle section into mean height.

78.—Cylindroid; its bases ellipses at right angles to each other.

Capital or base of elliptic column, connecting link between conduits; metallic envelope or tube flattened at ends in opposite directions.

Factors of middle section, arithmetic means between those of the bases. Lateral surface developed, a **plane trapeziform figure** of area equal to periphery of middle section into mean height, page 51 of "Key."

79.—Cylindroid or prismoid; its bases an ellipsis and a line.

Ridge roof to elliptical building or tower; a hut, camping tent, a strainer or filter; a finial or other ornament.

Middle section, a **mixtilineal figure** with factors, arithmetic means between those of bases. For area of middle section, page 57 of "Key." Lateral surface developed, a **plane trapeziform fig.**; its base, convex; its opposite base, angular. Area equal circumference of middle section into mean height.

80.—A compound solid; a cylinder and a cone. For cylinder, see No. 61, class VII.; for cone, see No. 81, class IX. The developed surface of a right cone is the **sector of a circle**. For area, see "Key to Ster." page 42.

A tower or other building, a hut, tent, or camp with conical roof; a hay rick, canister, finial; reversed: a cauldron, cistern, tub, filter, etc., etc.

CLASS IX.

Right and inclined Cone, Frusta, Angulae, etc.

81.—A right cone or infinitary pyramid.

Roof of tower, spire, finial or other ornament, pile of shot or shells, cornet, filter or strainer, funnel, etc.

Its base, a **circle**; its opposite base, a **point**; its middle section, a **circle** equal in area to one quarter that of base. Its lateral surface developed, the **sector of a circle**. For area of circle, see tables II., III., IV., "Key to Ster."

82.—Frustum of a right cone, considered as a prismoid.

A tower, quay, pier, base or capital of a column, flat roof to tower, component portion of a spire, a salting tub, etc.; reversed: a butter firkin, a tub or vat in a brewery or distillery, etc., a drinking goblet, bucket, pail, dish, basket, lamp shade; a vessel of capacity, the plug of a stop cock, etc., etc.

Its opposite and parallel bases and middle section, **circles**; its lateral surface developed, the **sector of a circular ring**, or a **curved trapezium**. The diameter of middle section an arithmetic mean between those of the opposite bases. For area of bases and section see "Key to Ster.," page 38; for lateral surface, page 43. Tables of areas of circles to eighths, tenths and twelfths, II., III., IV.

83.—Inclined or oblique cone.

Loop hole in a wall, the liquid or fluid substance in a conical vessel inclined to the horizon; a finial or ornament adapted to a raking cornice or pediment, etc.

Its base and middle section, **similar ellipses**—the latter equal in area to one quarter the former; the upper base, an **apex** or **point**; lateral surface developed an **irregular sector**, which, for computation of area, divide into triangles.

84.—Frustum of inclined cone.

Unequally splayed circular opening in a wall; a coal scuttle; reducer or connecting link between two conduits of different diameters laid eccentrically, etc.

Its opposite and parallel bases and middle section, **similar ellipses**; its lateral surface developed **portion of an eccentric annulus**, art. 39, page 43 of "Key to Ster." Diameters of middle section, arithmetic means between those of bases.

85.—Flat or low cone.

Roof to tower or circular construction; apex, a **point**; its middle section, a **circle** equal in area to one quarter that of base; its lateral face developed in a plane, the **sector of a circle**. For area of circle, see tables II., III., IV., of "Key to Ster.;" for sector, see page 42 of same.

Its base, a **circle**; opposite base or apex, a **point**; its middle section, a **circle** equal in area to one quarter that of base; its lateral face developed in a plane, the **sector of a circle**.

For area of circle, see tables II., III., IV., of "Key to Ster.;" for sector, see page 42 of same.

86.—Frustum of a low or surbased cone.

Flat roof to a pavillion, tower, etc.; a hat, the cover of a vessel of capacity; an unfinished or truncated pile of shot or shells; a lamp shade; a finial or other ornament; the bottom, base, top or other component section of a compound solid, as of No. 100; reversed: a dish, pan, saucer, cauldron, cistern, etc.

Its opposite bases and paralld middle section or intermediate base, **circles**; diameter of middle section, an arithmetic mean between those of the opposite bases; the lateral area developed in a plane, the **sector of a circular annulus**.

For areas of circles, see tables II., III., IV. of "Key to Ster.," sector, page 43 of same.

87.—Parabolic conic ungula by a plane parallel to side of cone.

Lunette to a circular headed opening in a wall and sloped ceiling; liquid in a closed conic vessel inclined to the horizon.

N.B.—For ratio of cord of middle section or segment to that of base, see "Key to Ster.," page 143, where it is shown that the squares of the cords are proportional to the abscissae.

The base, the **segment of a circle**; the opposite base, a **point**; the middle section, the **segment of a circle**; the plane of section a **parabola**. For areas of segment, see "Key to Ster.," page 44 and table VIII.; for area of parabola, page 54 of same. The lateral surface developed an **approximate sector of a circle**. The height or versed side of middle section segment is half that of base.

88.—Frustum of parabolic conic ungula by a plane parallel to base of cone.

Splayed opening or embrasure to a segment-shaped window or loop hole in a wall; lunette to opening in sloped ceiling terminating in a vertical surface; liquid in a closed vessel in the shape of the frustum of a cone, No. 82, when inclined from the vertical.

For cord of middle segment, measure solid or compute by page 143 of "Ster."

The parallel bases and middle section, **segments of a circle**; the lateral plane face or figure, the **zone of a parabola**, for area of which see "Key to Ster.," page 55, art (66); the developed conical surface, an **approximate sector of a circular annulus** or, more correctly, a **trapezium with curved concentric or parallel bases**, for area of which see note page 29, "Key to Ster.," For area of segment, table VIII., and page 44 of same.

89.—Frustum of a right elongated cone.

Shaft of Grecian column, tapered post, high tower or chimney shaft, funnel, pipe reducer, speaking trumpet or horn, plug of a stopcock or tap, deep drinking goblet, or other vessel of capacity large or small, shaft of a gun, component portion of many compound solids, etc.

Like No. 82, its opposite and parallel bases and middle section **circles**; diameter of middle section equal to the half sum of those of the bases; the developed lateral surface, the **sector of a concentric annulus**.

For areas of circles to eighths, tenths and twelfths, see tables II., III., IV., of "Key to Ster.," for that of sector, page 43 of same.

90.—A compound solid, composed of or decomposable into the frustum of a right cone and the segment or half of a sphere or spheroid.

For nature and areas of bases and middle section of the component frustum or a cone and of its lateral surface, see Nos 82 and 89.

For areas of bases and middle section

May represent a piece of ordnance, a deep conical vessel with hemi-spherical hemi-spheroidal or segmental bottom or top to it.

For hemi-sphere, hemi-spheroid, or segments thereof, greater or less than half, see classes 18, 19, 20.

For diameter of middle section in segment of spheroid, see "Key to Ster.," pages 139 and 140, where $AB : CD :: \sqrt{Ao \cdot oB} : oM$ and $CD : AB :: \sqrt{Co \cdot oD} : oM$, or, the rectangle under the required radius and either axis of the spheroid is equal to that under the square root of the rectangle or product of the abscissas of the first axis and the other axis.

of hemisphere or hemispheroid or of the segment of either, greater or less than a hemisphere, see tables II., III., IV. in "Key to Ster."

For diameter of middle section in hemisphere or in segment thereof, see "Bail-largé Geometry," par. 539 or Key to Ster.," par. 154, where $oa = \sqrt{Co \cdot oD}$, and $oD = \text{diam. } AB \text{ minus versed sine } oC$; or, the square of the half cord equals the rectangle under the versed sine and remainder of the diameter; or, may be obtained directly by measuring the solid.

CLASS X.

Conic Frusta and Ungulae, etc.

91.—Conic wedge or central ungula of a cone by planes drawn from opposite edges of the base to meet in the axis of the cone.

Ridge roof to a tower, splayed opening or embrasure to a long narrow vertical loop hole in a wall; component section of compound solid as of a cone and cylinder or of cones having their bases or apices in opposite directions.

92.—Frustum of a conic wedge or of the central ungula of a cone by a plane parallel to base; or, may be considered the frustum of a right cone, laterally and equally truncated on opposite sides.

Arched and splayed embrasure in a wall, component portion of a compound solid.

93.—Lateral elliptic ungula of a cone, by a plane passing through edge of base.

Splayed embrasure to elliptic opening in wall and through sloped roof or ceiling, etc.

The base, a **circle**; the parallel upper base, an **arris or line**; the middle section parallel to bases, the **zone of a circle**; the lateral plane faces **equal segments of equal ellipses, each greater than half**; the curved or conical faces developed, equal **curvilinear triangle**.

For areas, see pages 38, 46, 53 and 57, and tables II., III., IV., of "Ster." For area of zone, see table IX. of same.

The base, a **circle**; the opposite and parallel base, a **zone of a circle**; the middle section, a **zone**; the lateral plane faces, **equal segments of equal ellipses**; the developed conical surfaces resolvable into **trapeziform figures**.

For area of trapezium, page 29, "Key to Ster."

Its base, a **circle**; its upper or opposite base, a **point**; its middle section parallel to base, the **segment of a circle**; its plane face an **ellipsis**; its conical surface developed a **concavo-convex figure** like h, page 97 of "Key to Ster."

94.—Lateral elliptic conic ungula, by a plane passing within the base.

The liquid in an inclined conical vessel, lunette head of opening in sloped roof or ceiling; base of structure rising from an inclined surface, roof, pediment, etc.

For area of parabola see key to Ster., page 54; for area of hyperbola, page 55, or figure e, page 57; for ellipsis, page 51 and 53.

95.—Central ungula of cone or conic wedge, by planes through opposite edges of upper or lesser base and meeting in the axis of the cone.

An embrasure, etc., etc.

The plane lateral faces, segments of ellipses if cutting planes more inclined to base than side of cone; if less, hyperbolas; if equally, parabolas.

96.—Frustum of conic wedge, No. 85, by a plane parallel to the base.

An embrasure; a reducer or connecting link between a rectangular and circular conduit, etc.

97.—Concave ungula of a cone or a conical recto-concave wedge.

Lunette of circular headed opening in wall, reaching through vaulted, groined or arched ceiling; cone scribed to cylindrical surface, or to a shaft of elliptical section.

98.—Portion of frustum of right cone, by a plane through both bases.

Splayed segment headed opening in wall, liquid in closed tub lying on its side; base or capital of half column against sloped wall; component section of base or capital of clustered, Gothic or other column.

99.—Lateral conic ungula or wedge, by a plane through edge of lesser base of frustum.

The base, a segment of a circle; the upper base, a point; the middle section, a segment of a circle; the plane lateral face, the segment of an ellipsis; the developed conical surface as in No. 87 or 94. If the cutting plane be parallel to side of cone the face will be a parabola; if at an angle greater than side of cone to base, hyperbola; if less, an ellipsis.

Bases and sections same as No. 91; developed conical surface, a concavo-convex triangle computable as per page 57 of "Key to Ster."

The lateral plane faces, equal segments of equal ellipses, equal parabolas or equal hyperbolas, as case may be.—See No. 94.

Its base, a circle; other base and middle section, zones of circles, for areas of which see "Key to Stereometicon, table IX.

The base, the segment of a circle; the other base, a point or curved arris; its intermediate base or section, or its bases or sections if divided for computation of cubical contents, segments of circles. Its sides like No. 94.

Its parallel end bases and middle section, segments of circles; its conical surface developed a figure of trapezium form, having parallel or concentric arcs of circles for its bases; its plane face, the zone of an ellipsis or of a parabola or hyperbola according to inclination of cutting plane.

Its base, a circle; opposite base, a point; intermediate section a segment of a circle; its plane face an

Embrasure, liquid in inclined conical vessel, section of conical elbow or mitre, base of chimney stack to sloped roof. May be treated also as lying on its lateral plane face.

ellipsis, its conical surface developed a concavo-convex figure like *g* or *h*, page 97 of *Ster.* but with concave base. Treat on circular base as easier of computation.

100.—A compound solid composed of decomposable or resolvable into **two conic frusta and a low or flat cone.**

May represent a covered dish, a basket or hamper, a vase, a finial or other ornament, an urn, a cauldron on a stand, etc., etc.

All its areas to be used in computation of solid contents or capacity are **circles**, and can be measured to eighths, tenths or twelfths of an inch or other unity, and the areas found by mere inspection in tables II., III. and IV. at end of Baillargé's "Key to *Ster.*"

DIVISION 2.

Solids of double curvature, or of which the surfaces are not capable of development in a plane.

CLASS XI.

Concave Cones, Frusta and Ungulae.

101.—Right concave cone or spindle.

Camping tent; roof of tower, pavilion, hut, etc.; spire, funnel, strainer, trumpet; finial or other ornament.

May be decomposed into two or more frusta by planes parallel to base, to admit of more accurate determination of solid contents.

Its base and parallel sections, **circles**; its upper or opposite base, an apex or **point**. Its lateral surface not capable of development in a plane or into a sector of a circle as is the case with a regular right cone, but may be readily and very approximately computed by division into **continuous trapeziums** by lines parallel to circumference of base. See "Key to *Ster.*," page 96.

102.—Frustum of a right concave cone between parallel planes.

Illustrative of most of the objects mentioned in No. 82, which see.

For more accurate computation of contents, divide into two sections or more, according to greater or lesser curvature of the solid, and treat each section as a separate prismoid and add the results.

Its bases and parallel sections, **circles**. Intermediate diameters not, as in No. 82, arithmetical means between those of the opposite or end bases, but must be measured or computed. Lateral area may be conceived as made up of a series of super or juxta-posed **continuous trapeziums**.

103.—Inclined concave cone.

Finial, or ornament on a raking cornice; liquid in an inclined vessel, etc., as for No. 101, may be decomposed by imaginary planes parallel to base into two or more sections or slices, so that

Its base and section, **approximate ellipses of slight excentricity** or **ovoid figures**; its other base, a **point**.

In developing the lateral surface into a series of **continuous trapeziums**, the lines are not as in the right cone

slant side of each may be sensibly a straight line. See page 103, par. 139, "Key to Ster."

parallel to base or to circumferences of parallel sections, but are drawn equidistant from the apex, thus leaving at the base a figure like h, page 57 of "Key to Ster."

104.—Frustum of oblique concave cone between parallel planes.

Representative of same as No. 84.

Its bases and sections parallel thereto, approximate ellipses or ovoid figures. See remarks to No. 102.

105.—Flat or low concave cone.

Representative of many of the objects mentioned in No. 85.

Its bases, a circle and a point; section, a circle; lateral area reducible to continuous trapeziums, par. 126, "Key to Ster."

106.—Frustum of flat or low cone.

Representative of objects under head of No. 86.

Its bases and section, circles, for areas of which see tables II., III. and IV. of "Key to Ster.," to eighths, tenths and twelfths of inch or other unity.

107.—Ungula of concave cone by a plane through outer edge of base.

See No. 92, as to what it represents, etc.

See No. 92. Lateral surface reducible to trapeziums and triangles.

Base and sections, ovoidal figures; areas, page 57 of Key.

108.—Ungula of concave cone by a plane cutting the base.

See No. 93 as to what it represents, etc.

Bases and section, segments of circles; upper base, a point. Lateral surface as No. 107.

109.—Ungula of hollow cone by a plane through edge of lesser base of frustum.

See No. 99, base of chimney stack to a sloped roof.

Base, a circle; opposite base, a point; middle section, the segment of a circle; lateral area, trapeziums and triangles.

110.—Frustum of (No. 109) ungula by a plane parallel to base.

See Nos. 98, 116, 126.

Base or capital of a column, or base of chimney shaft, etc., on or outside of sloped roof or gable.

Its base, a circle; other base, a segment of a circle; its middle section parallel to bases, also a segment. For areas of segments of circles, see "Key to Ster.," table VIII., or rules, page 44 of same.

Paraboloid or Parabolic Conoid, Frusta and Ungulae, etc.

111.—Right paraboloid or parabolic conoid.

Dome, hut, hive, roof, finial or other ornament, shade, globe, cover, hood, cowl, etc.; reversed: a filter, cauldron, or other vessel of capacity, the bowl of a cup or drinking goblet, etc., etc.

Its base and middle section, **circles**; its opposite base or apex, a **point**; its lateral surface resolvable into a small circle at apex, and **continuous trapeziums**. The squares of its intermediate diameters, proportional to abscissae. See "Key to Ster.," page 96.

112.—Frustum of right paraboloid, between parallel planes.

Represents mostly the same objects as the frustum of a cone, No. 82.

See page 142 "Key to Ster."

End and middle bases, **circles**; squares of diameters proportional to abscissae. For areas of circles, see "Key to Ster.," tables II., III. and IV.

113.—Oblique paraboloid.

"Key to Ster.," page 142.

Liquid in a parabolic vessel inclined to the horizon, metal in an inclined crucible, finial or ornament on an inclined or raking molding or pediment, etc.

Its base and middle section, **similar ellipses**; its opposite base or other end, an apex or **point**. For areas of ellipses see "Key to Ster.," page 51; for lateral area see No. 103.

114.—Frustum of oblique paraboloid between parallel planes.

Represents same as frustum of inclined cone No. 84, "Key to Ster.," page 142.

Its bases and middle section, **similar ellipses**; for areas of which see "Key to Ster.," page 51. For lateral area, see No. 103 or reduce to **trapeziums** by lines from base to base.

115.—Parabolic wedge or central ungula of paraboloid.

See No. 91.

Lateral or paraboloidal surface capable of approximate development. See No. 91.

116.—Portion of a paraboloidal frustum, by a plane through its greater base and edge of other or opposite base.

See No. 98 as to what it represents. Also, base of chimney stack, partly on a horizontal and partly on an inclined base, or sloped roof, etc.

Its lesser base, a **circle**; opposite base, the **segment of a circle**; middle section, also a **segment**. Its lateral plane face, the **segment of an ellipsis**. This face would be a parabola if angle of face equalled that of side; if greater, a hyperbola.

117.—Lateral ungula of paraboloid.

Very similar to No. 92, as to what it represents.

Its base, a **circle**; opposite base, a **point**; middle section, the **segment of a circle**. Its plane face an **ellipsis**.

118.—Lateral ungula of paraboloid; elliptic, parabolic or hyperbolic, according as plane of section cuts the base at an angle less than, equal to, or greater than that of the side and base.

Its base, the segment of a circle; its middle section, a segment; its upper or opposite base a point; its plane face, the segment of an ellipsis, parabola or hyperbola, according to angle of plane of section.

119.—Obtuse elliptic ungula of a paraboloid, by a plane through edge of lesser base of frustum.

Base of chimney stack, etc., to sloped roof; base of a vase, statue, etc., on a pediment; a lunette, scoop, etc.

Its base, a circle; middle section, a segment; other base, a point; its plane face, an ellipsis. For areas of segments of circles, table VIII. of "Key to Ster." For area of ellipsis, page 51 of same.

120.—Frustum of a paraboloid between non-parallel bases.

"Key to Ster.," page 145.

Lunette through a vertical wall and inclined ceiling, etc. For computation of solid contents decompose into a frustum with parallel bases and an ungula by a plane parallel to base, through nearest point of upper base.

Its factor areas, circles and a segment; its plane face, an ellipsis. For areas of segments of circles, table VIII. of "Key to Ster." Area of circle, tables II., III. and IV. of same; ellipsis, page 51 of same; lateral area, page 95; solidity, page 145 of same.

CLASS XIII.

Hyperboloid or Hyperbolic Conoid, Frusta and Ungulae, etc.

121.—Right hyperboloid or hyperbolic conoid.

Page 146, "Key to Ster." Representative of same as No. 111.

For intermediate diameter or that of middle section, see "Key to Ster.," page 147, 3rd line, or by direct measurement.

122.—Frustum of right hyperboloid.

Representative of same, nearly as No. 112 and 82.

Except for diameter of middle section, same as No. 112, or the diameter may be measured directly.

123.—Oblique hyperboloid.

See "Key to Ster.," page 146. Representative of same, as No. 113.

Same as No. 113, except for diameter of middle section for which see "Key to Ster.," page 147, line 3, or the diameter may be measured.

124.—Frustum of oblique hyperboloid.

Representative of same, nearly as Nos. 84 and 114.

Same as No. 114, except for diameter of middle section for which see "Key to Ster.," page 147, line 3, or may be had by measurement.

125.—Hyperboloid wedge or central ungula.

Similar solid to No. 95 of a cone and representative of same objects.

Except for diameter of middle section, same as No. 91 or 95. For area of zone, see "Key to Ster.," page 46 or table IX. of same.

126.—Ungula of hyperboloid by a plane through edge of base.

For solid content, treat as prismoid or by par. 185 of "Key to Ster."

Solid similar to No. 93 of cone, or to No. 117 of paraboloid.

Its base, a circle; middle section, the segment of a circle; other base, a point. Plane lateral face, an ellipsis, its lateral surface of double curvature, as all such figures are not capable of development, but reducible as required.

127.—Frustum of hyperboloid wedge.

Similar to No. 116 of paraboloid. Base of chimney stack, etc., resting partly on a sloped roof.

Bases same as in No. 116. Lateral area develops into trapeziums by lines parallel to bases. For areas of circles, segments, zones, see tables of "Key to Ster."

128.—Ungula of hyperboloid by a plane through base.

Similar to No. 118 of paraboloid.

Bases and section same as No. 118 of paraboloid. See table VIII. of "Key to Ster.," for areas of segments.

129.—Frustum of hyperboloid wedge, or of central ungula of hyperboloid.

Similar to No. 92 of cone.

Same as No. 92. For areas of circles to eighths, tenths, and twelfths, see tables II., III. and IV. of "Key to Ster." For area of zone, see table IX. of same. Lateral surface decomposable into trapeziums.

130.—A compound solid. Two equal frusta of cone or conoid, base to base.

Illustrative of a keg or cask, barrel, hogshead, etc., of any size or shape.

Treat one-half of solid as Nos. 82, 112, 122, and double the result.

See "Key to Ster.," fig. on page 155, for mode of measuring half-way diameter, when the half solid is not the frustum of a cone, but that of a conoid or of an ellipsoid or spheroid. When of a cone middle diameter equal to arithmetic mean of end diameters.

CLASS XIV.

Sundry Solids.

131.—Three axed spheroid.

See "Key to Ster.," page xxxix. May for measurement be supposed to lie or stand on either of its sides or apices.

Representative of a pebble, a beam, spindle, torpedoe, a shell fish, a flattened ellipsoid, etc., etc.

All its sections, ellipses; all its parallel sections, similar ellipses. For areas of ellipses, "Ster.," page 51. Lateral area, see general formula, page 95, "Key to Ster." Or, as with the spheroid, suppose the surface divided as a melon is or orange into ungulae, terminating in apices or poles of the fig.

132.—An ovoid or solid of the shape of an egg.

Divide into two or three sections and treat separately as conoid, segment of sphere or spheroid, and frustum of conoid.

133.—Circular disc with rounded edge.

Treat as a compound solid, to wit: a flat or low cylinder, and a ring semi-circular or segmental in section. Add the results.

134.—Twisted prism.

Portion of a circular stairs rail, a twisted pillar or column, spiral ornament, etc.

135.—A compound solid.

Two frusta of cones, their lesser bases joined.

A windlass, spool, handle, shaft, axle-tree, etc.

136.—A compound solid.

Two frusta of hollow cones joined by their lesser bases.

A windlass, spool, handle, shaft, axle-tree, etc.

137.—Compound solid.

Two frusta of concave cones joined by their greater bases.

A windlass, shaft, axle-tree, etc.

138.—Compound solid.

The segment or half of an elongated or prolate spindle, No. 151, and the segment or half of an oblate spindle, No. 141, or the segment of a sphere or spheroid, classes XVII. and XIX., a buoy, etc.

139.—Compound solid like the last with hollow cone instead of spindle.

A finial or other ornament, a cul-de-lampe or pendant.

All parallel areas perpendicular to longer or fixed axis, **circles**, which find ready calculated for all sized diameters to eighths, tenths, and twelfths of an inch, or other unity of measure, tables II., III., and IV., of Ster. For lateral area, see page 96 of same.

For cylinder, see No. 61. For ring compute area of section thereof as **semi-circle** or **segment**, and multiply into circumference. For area, mean circumference of ring into circumference of section.

Its bases and sections similar and equal figures. The lateral surface of each face can be developed in a plane, a **trapezium** or **rectangle**.

Treat half the solid as the frustum of a cone, and double the result, either for solid content or area of figure.

Treat one half the solid as frustum of cone No. 102, and double the result.

Lateral area resolvable into continuous trapeziums.

Treat half the solid, and double the result. For areas of circles, see tables II., III. and IV. of Ster.

Sections perpendicular to axis, **circles**; Area resolvable into **continuous trapeziums**, a **circle** and the **sector of a circle**. The circle at apex of segment of sphere or spheroid; the sector at apex of spindle. See page 55 of "Key to Ster."

Sections perpendicular to axis, **circles**. Lateral surface, **continuous trapeziums**, a **circle**, and the **sector of a circle** at apex of cone.

140.—Compound solid: the frustum of a sphere or spheroid and a hollow cone. Bases and sections, **circles**. Lateral surface resolvable into **continuous trapeziums**. See general formula, page 95 of "Key to Ster."

A Moorish dome, a minaret, chimley of a coal oil lamp, a decanter, a vase, a pitcher.

CLASS XV.

Oblate or Flattened Spindle, Frusta, Segments, Sundry.

141.—Oblate spindle, as two equal segments of sphere or spheroid base to base. Treat one half as segment of sphere or spheroid, and double the result. See classes 17 and 19.

A quoit, etc.

142.—Semi-oblate spindle by a plane parallel to fixed axis. Treat its two halves together as one segment of sphere or spheroid. See classes 17 and 19.

Floating caisson to entrance of dock, etc.

143.—Middle frustum of oblate spindle. The bases and middle section each a **double segment of a circle or ellipsis, or two segments thereof, base to base**. Table VIII., "Key to Ster."

Fixed caisson or coffer-dam. Treat as prismoid.

144.—Lateral frustum of oblate spindle, between planes parallel to fixed axis. The bases and section half-way between them, **double segments of circles or ellipses**, for areas of which see table VIII., "Key to Ster.," and page 53 of same.

A flat-bottomed boat or other sailing vessel or a caisson, etc.

145.—Lateral frustum of oblate spindle truncated at one end. Bases and middle section, **double segments, base to base, of circles or ellipses truncated at one end**. For areas, see page 57 "Key to Ster."

A flat-bottomed boat or other sailing vessel.

146.—Lateral frustum of oblate spindle truncated at both ends. Bases, **double segments of circles or ellipses truncated at both ends**. Divide into trapeziums and compute areas by page 57 "Key to Ster."

A flat-bottomed boat or pontoon, a scow, lighter, etc.

147.—Quarter of an oblate spheroid, No. 181. Its base and middle section, **semi-circles**, if treated on its broader base; if on its lesser face, its base and middle section, **semi-ellipses**. On whatever base it stands, treat as if on broader

The arched ceiling, roof or vault of the apsis of a church or half-groined ceiling of a circular apartment. On its

lesser base, the head of a shallow niche in a wall, etc.

148.—A compound body, a cone, and the segment of a sphere or spheroid.

A buoy, covered filter, etc.

base, it being easier to compute circles than ellipses.

Treat separately as cone No. 81, and as segment of sphere, No. 173, or of spheroid No. 182.

149.—Elliptic ring, or may be called an eccentric ring.

Treat as circular or cylindrical ring, taking for bases its least, its greater, and its mean sections; and for length the mean of the inner and outer circumferences.

Compute half of solid as the lateral frustum of a half-prolate spindle or the frustum of an elongated cone. The solid may be conceived to be formed of the middle frustum of an elongated spindle bent till its ends meet.

150.—Compound solid: a cylinder and the segment of a sphere or spheroid.

A mortar, a tower with domed roof, a hall or room with groined ceiling, a hut, hive, hood.

For area of sphere or spheroid, see page 95 "Key to Ster.," or page 105, 110, 124, Ex. 3. Areas of circles tables II., III. and IV. of same. Half-way diameter in segment of circle or sphere a mean proportional between abscissae of diameter.

CLASS XVI.

Prolate or Elongated Spindle, Frusta, Segments, etc.

151.—Prolate spindle.

A shuttle, a torpedoe, a cigar, a sheath, case, etc.

Its sections perpendicular to axis, circles. Decompose its lateral area into continuous trapeziums and a sector.

152.—Semi-prolate spindle by a plane through its greater or fixed axis.

A boat or sailing vessel, a canoe, etc.

For solidity, compute planes perpendicular to fixed axis, as segments of circles, while the sections parallel thereto are not so readily computed.

153.—Semi-prolate spindle by a plane perpendicular to fixed axis.

A hut, roof, filter or vessel of capacity a minaret or finial.

For greater accuracy, divide into a frustum and segment, compute and add cubical contents. Areas of bases, tables II., III. and IV. of "Key to Ster."

154.—Middle frustum of prolate spindle between planes perpendicular to fixed axis.

A cask or keg, puncheon, hogshead, etc.; see page 155 "Key to Ster."

See page 149 of "Key to Ster.," and for lateral surface, page 95 of same. See page 155 of same. Bases and sections, circles, tables II., III. and IV. of "Key to Ster."

155.—Semi-middle frustum of prolate spindle.

The liquid in a cask lying on its side, a boat with truncated ends. Compute as No. 154 and take half.

Bases and middle section, **semi-circles**, see page 160 of "Key to Ster." Lateral surface decomposable into **trapeziums**.

156.—Lateral frustum of prolate spindle by planes parallel to fixed or longer axis.

A flat-bottomed boat or other sailing vessel.

Treat as prismoid, the greater base, a **double segment of a circle**. The other base and section, **oval figures** for areas of which see page 57 of "Key to Ster."

157.—Eccentric frustum of a prolate spindle by planes perpendicular to fixed or larger axis of solid.

The shaft of a Roman column. Compute each frustum from centre and add the results.

Its bases and sections, **circles**. For areas of which to eighths, tenths and twelfths of inch or other unit of measure, see tables II., III. and IV., "Key to Ster."

Its lateral surface decomposable into **continuous trapeziums**, or nearly equal to length of side into mean circumference.

158.—Middle frustum of elongated spindle by planes perpendicular to fixed or longer axis.

The shaft of a windlass, a drum or pulley, a cigar, torpedoe, etc.

Its bases and sections, **circles**, for areas of which see "Key to Ster.," page 38, or tables II., III. and IV. of same.

Lateral area equal nearly length of curved side into mean of circumferences.

159.—A curved half-spindle or cone.

A horn, powder flask, tusk or tooth of an elephant, etc., a supporting bracket from face of wall.

Base and sections, **circles or ellipses of slight eccentricity**. Lateral area decomposable into **continuous trapeziums** and sector at apex.

160.—Frustum of a prolate spindle between non parallel bases.

Decompose into a **frustum with parallel bases and an ungula** by a plane through nearest point of one of the bases.

Base and sections parallel thereto, **circles**; base of ungula a **circle**; middle base of ungula, a **semi-circle**; apex of ungula or opposite base, a **point**; lateral surface, **continuous trapeziums**, and a fig. like h, page 57 "Key to Ster."

CLASS XVII.

Sphere, Segments, Frusta and Ungulae, etc.

161.—The sphere.

A billiard or other playing ball, the ball of a vane or steeple, spherical shot and shell, school spheres, lamp globe or well, component part of compound solid,

The opposite bases, **points**; the middle section, a **circle**. The area of surface admits of approximate development into a series of equal figures in the shape of the longitudinal section of a prolate

etc. Solid content may be had by computing one of the component ungulae and multiplying into the number thereof.

162.—A hemisphere.

A dome, arched ceiling, globe, shade, cover, hut, hive, etc. ; reversed : a bowl, cauldron, copper, vase, etc.

Contents more easily computable as half of those of a whole sphere, where there is no intermediate diameter to calculate or measure.

163.—Segment of a sphere less than a hemisphere.

Representative of same objects as No. 162, cover or bottom of a boiler. Solid contents also equal to one of the component ungulae into the number thereof.

164.—Segment of sphere, greater than a hemisphere.

Representative of same as No. 162, and of a Moorish or Turkish or horse-shoe dome.

165.—Middle frustum of a sphere.

Base, capital or middle section of a column or post, a puncheon, hogshead, crusher, roller, lamp shade, etc., etc.

166.—Lateral frustum of sphere.

Base or capital of column, coved ceiling, cauldron, dish, soup plate, saucer, etc. Radii of bases and sections proportional to square roots of rectangles of portions into which such radii or ordinates divide the diameter of the sphere of which the solid forms a part.

167.—Spherical wedge or central ungula of a sphere by planes from opposite edges of base of hemisphere to meet in apex.

Component portion of a compound solid.

spindle, or of double segments of a circle, base to base.

Surface equal to four great circles or to four times that of a great circle.

Its base, a circle ; opposite base, a point ; its middle section, a circle, the half diameter of which equals the square root of the rectangle under the versed and su-versed sines or portions of the diameter of the sphere. The lateral area equal to two great circles of the sphere.

Base and section, circles ; other base, a point ; radius of middle section for area thereof, equal to root of rectangle of parts into which it divides the diameter of the sphere of which the segment forms part. For lateral area see "Key to Ster.," page 110, or General Formula, page 95.

Its base and section circles ; other base a point ; radius of middle section the root of rectangle of parts into which it divides diameter of sphere. Lateral area, see "Key to Ster.," pages 117 and 123.

Bases, equal circles ; middle sections, a circle ; see tables of areas of circles to eighths, tenths and twelfths of an inch or other unity of measure, II., III. and IV. of "Key to Ster."

Bases and section, circles ; lateral area resolvable into continuous trapeziums ; or lateral area may be had very nearly at one operation, if the frustum be low or flat and that its lateral curvature be not considerable.

Its base, a circle ; opposite base, a ridge, or axis, or line ; middle section, the zone of a circle ; its plane faces, circles ; and lateral area resolvable into trapeziums and triangles.

168.—Frustum of a spherical wedge or central ungula between parallel planes.

Component portion of compound solid.

Base, a circle; other base and middle section, zones of circles. For areas of zones, see table IX., "Key to Ster."

169.—Spherical pyramid, obtuse-angled and triangular.

Illustrative of the tri-obtuse-angular spherical triangle, and of the fact that the sum of the angles of a spherical triangle, may reach to six right angles, when each of the component angles increases to 180° .

Base, a spherical triangle having three obtuse angles; apex or opposite base, a point; middle section, a similar tri-obtuse angular spherical triangle, and whose area is equal to one-quarter that of base, its factors being halves of those of base, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

170.—Frustum of sphere between non-parallel bases.

Elbow or connecting link between two portions of a rail or bead; base of a vase or other ornament on a raking cornice.

Decompose into frustum and ungula of a sphere by a plane parallel to one of the bases and passing through nearest point of other base, or more readily and exactly, compute whole sphere, and deduct segment.

CLASS XVIII.

Spherical Ungulae, Sectors, Pyramids and Frusta.

171.—Quarter-sphere or rectangular ungula of a sphere.

Domed roof to a semi-circular plan, vault of the apsis of a church, head of a niche, "Key to Ster.," page 117.

Compute as a whole sphere, and divide by 4, or treat as an ungula. See opposite par.

On its base: one base, a semi-circle; opposite base, a point; middle section, the segment of a circle. On end: each of its opposite bases, points; its middle section, the sector of a circle. Only one area to compute, and easier and quicker than a segment.

172.—Acute-angled spherical ungula.

Component portion of the ball of a vane or steeple; natural section of an orange, or of a ribbed melon, section of a buoy, cauldron, etc., elbow of two semi-cylindrical mouldings, etc., at an obtuse angle.

Its opposite bases, points; its middle section, the sector of a circle; the spherical surface, the component of a hollow metallic or other sphere or spherical vessel, or of the covering for a racket or other playing ball, etc.

For spherical area see "Key to Ster.," page 117.

173.—Obtuse-angled ungula of a sphere.

Head of niche reaching into a sloped ceiling; elbow of two half-beads at an acute angle, etc.

Opposite bases, points; middle sections, the sector of a circle; its plane faces, semi-circles. Spherical area, page 117 "Key to Ster."

174.—Spherical sector or cone, or, to avoid computing spherical areas, may be treated as a compound body, a cone and the segment of a sphere.

A buoy, a finial or ornament, a top, etc., a covered filter. For areas of circles see tables II., III. and IV. of "Key to Ster."

175.—Frustum of a spherical sector between parallel spherical bases.

Portion of a shell or bomb or hollow sphere. To avoid computing spherical areas, treat as frustum of cone, adding greater and deducting lesser segment.

176.—Hexagonal spherical pyramid.

Its base illustrative of a spherical polygon, page 127 of "Key to Ster."

Component portion of a solid sphere or ball; keystone of a vault, finial or other ornament; decomposable for computation into six equal triangular spherical pyramids, "Key to Ster.," page 129. See rule for spherical areas at end of this pamphlet.

177.—Frustum of hexagonal spherical pyramid between parallel bases.

Keystone of vault. Component portion of hollow sphere. Surfaces illustrative of similar spherical polygons. Height of solid equal slant height of side.

178.—Half-quarter or one-eighth of sphere or tri-rectangular spherical pyramid.

Termination or stop to chamfer on angle of wall or pillar.

Compute whole sphere and divide by eight.

179.—Acute equilateral triangular spherical pyramid.

Its base illustrative of the equilateral spherical triangle.

Its base, a spherical segment; the other base, a point; middle section, a spherical segment concentric to the base and equal in area one quarter of base; its height equal to radius of sphere, its lateral face developed, the sector of a circle. See "Key to Ster." page 110.

Its bases and middle section parallel thereto, concentric and similar segments of spheres of corresponding radii. Its height, the length of slant side. Solidity also equal to difference between whole and partial spherical sectors.

Its base, a regular six-sided spherical polygon; its middle section a figure similar to the last, and equal in area to one-quarter thereof; its opposite base, a point, the centre of the sphere of which it forms part. For area of base, see "Key to Ster.," page 127. For area of component spherical triangle of base, see page 123 of same. Its plane faces equal sectors of a circle.

Its bases and middle section, similar spherical polygons; factor of middle section, as in cone, an arithmetic mean between those of the bases. Its lateral faces, equal frusta of equal sectors of a circle, or concavo-convex trapeziums. See rule at end of this work.

Its base illustrative of the tri-rectangular spherical triangle, page 123 of "Key."

May compute for solid contents as the half of an ungula where only one area is required, that of a sector of a circle. See rule at end of this work.

Base and middle section similar equilateral spherical triangles, for areas of which, see "Key to Ster.," page 123, and rule at end of this work.

180.—Frustum of triangular spherical pyramid

Illustrative in its bases of similar spherical triangles. Keystone of a vault to a triangular plan.

Bases and middle section, **similar spherical triangles** whose areas are as the squares of the corresponding radii; or factors of middle section, arithmetic means between those of the opposite bases.

CLASS XIX.

Oblate Spheroid, Frusta and Segments.**181.—Oblate spheroid.**

Representative, in a less exaggerated ratio of its diameters or axes, of the earth and planets which are flattened at the poles or extremities of fixed axis and protuberant at the equator. An orange, lamp-shade, or globe, or bowl.

Treated perpendicularly to its fixed axis, its opposite bases are considered **points**, as in the sphere, a plane touching the solid only in a point; its middle section, a **circle**. If considered parallel to its fixed axis, its middle section, an **ellipsis**. For spheroidal surface or area, see No. 161.

182.—Semi-oblate spheroid by a plane perpendicular to its fixed or lesser axis.

Elliptical ceiling, dome, cauldron, basin, dish, vase, shade, globe, etc.

Base, a **circle**; opposite base, a **point**; middle section, a **circle**; for diameter of which, if not from direct measurement, see "Key to Ster.," page 139, line 10 and page 140, line 20.

183.—Semi-oblate spheroid by a plane parallel to its fixed or lesser axis.

Dome or ceiling to an elliptic plan; glass globe or shade, dish cover, hut, a trough, cauldron, etc.

Equal in area and solid contents to No. 182 and of easier and quicker computation, if considered such, the factors being **circles** instead of **ellipses**. As it stands, its base and middle section, **similar ellipses**.

184.—Segment of oblate spheroid, greater than half by a plane perpendicular to fixed axis.

Turkish, Moorish or horse-shoe dome or ceiling; a cauldron or copper, etc.

Its base and middle section, **circles**; opposite base, a **point**. Spheroidal surface **continuous trapeziums** and a **circle** at apex. For areas of circles, see tables II., III. and IV. of "Key to Ster." For factors of middle section, see No. 182.

185.—Middle frustum or solid zone of an oblate spheroid between planes perpendicular to fixed or shorter axis.

Representative of same as No. 165.

Opposite bases and middle section, **circles**; for areas of circles to eighths, tenths and twelfths of an inch or other unity, see tables II., III. and IV. of "Key to Ster." Spheroidal area, see page 95 of same.

186.—Middle frustum or solid zone of oblate spheroid by planes parallel to fixed or lesser axis of solid.

Its bases and middle section **similar ellipses**, for areas of which see page 51 of "Key to Ster." Spheroidal area, page 95 of same.

187.—Segment of oblate spheroid less than half, by a plane parallel to its fixed or lesser axis.

Representative of same as No. 183.

Its base, an **ellipsis**; opposite base, a **point**; middle section, an **ellipsis** similar to base. For factors of middle section, see No. 182.

188.—Lateral frustum of oblate spheroid by planes parallel to fixed or shorter axis.

Its opposite parallel bases and middle section, **ellipses**, for areas of which see "Key to Ster.," page 51.

Coved ceiling of elliptic plan; reversed: a boat, a scow, a vessel of capacity, etc.

Its spheroidal surface decomposable into **continuous trapeziums** of variable height.

189.—Half or segment of oblate spheroid by a plane inclined to axis of solid.

Its base and middle section, **similar ellipses**; its opposite base, a **point**; its spheroidal surface, **trapeziums**, with **ellipsis** at apex and a **curvilinear triangle** at base of shape similar to fig. h, page 57 of "Key to Ster.," or lateral area may be divided and computed as triangles.

Liquid or fluid in a semi-spheroidal vessel inclined from the vertical. Finial on a pediment or sloped surface.

190.—Frustum of oblate spheroid between non-parallel bases.

Bases and middle section of component frustum with parallel bases, **ellipses**; base of ungula, an **ellipsis**; middle section of ungula the **segment of an ellipsis**; its other base, a **point**.

Decompose into a frustum with parallel bases, and an ungula by a plane parallel to one base and drawn through nearest point of other base, or compute whole spheroid and deduct segments.

For factors of middle sections, see "Key to Ster.," page 139, line 10, and page 140, line 20, where $AB : CD :: \sqrt{Ao.oB} : oM$ and $CD : AB :: \sqrt{Co.oD} : oM$.

CLASS XX.

Prolate Spheroid, Frusta and Segments.

191.—Prolate spheroid.

Representative of a lemon, melon, cucumber, etc.; a case, sheath, etc.

The work of computation expedited by treating circles instead of ellipses, that is, areas perpendicular instead of parallel to fixed axis.

Its middle section perpendicular to fixed or longer axis, a **circle**; its opposite end bases, **points**. Spheroidal surface, **continuous trapezoids**, or a series of **double segments base to base** as the component ribs of a melon. May treat as plane segment with length of cord equal to semi-elliptical section.

192.—Semi-prolate spheroid by a plane parallel to fixed axis.

Vaulted ceiling to elliptic plan; reversed: a boat or other sailing vessel, a cauldron or vessel of capacity, etc., etc.

For solid contents and spheroidal surface treat perpendicular to fixed axis, where factors are **circles** or **semi-circles** instead of **ellipses**. For areas of circles, see tables II., III. and IV. of "Key to Ster."

193.—Semi-prolate spheroid by a plane perpendicular to fixed axis.

A hive, hut, roof or dome to circular tower or apartment; reversed: a copper or boiler.

Base, a **circle**; other base, a **point**; middle section, a **circle**. For radius of middle section, see formula given in No. 190, or at page 139, line 10, page 140, line 20 of "Key to Ster." Spheroidal area, see No. 191.

194.—Segment of prolate spheroid greater than half, by a plane perpendicular to fixed axis.

A hut, hive, dome, a cauldron or copper, etc.

Base and middle section, **circles**; its other base, an apex or **point**. Its spheroidal surface resolvable into **continuous trapeziums** and a **circle** at apex.

195.—Middle frustum or solid zone of prolate spheroid by parallel planes perpendicular to fixed axis.

A cask, keg, barrel, puncheon, hog's head, etc., "Key to Ster.," page 138.

End bases, equal **circles**; middle section, a **circle**. Unlike the middle frustum of a spindle, the solid contents of this solid are obtained exactly by treating the whole figure at once.

196.—Middle frustum or solid zone of prolate spheroid by parallel planes oblique to axis.

Opposite bases and middle section, **similar ellipses**. Spheroidal surface, **trapeziums** of which take mean height.

197.—Lateral frustum or solid zone of prolate spheroid by planes perpendicular to fixed axis.

Coved ceiling, base of column, etc.; reversed: capital of column, dish, basin, bowl, tub, hamper or basket, stew pan, cauldron or other vessel of capacity, etc., etc.

Bases and section, **circles**, for areas of which see tables II., III. and IV. "Key to Ster." For diameter of middle section, measure solid or compute by formula of page 139, line 10; page 140, line 20, where it is shown that the rectangle under the required radius, and either axis of the spheroid is equal to that under the square root of the rectangle or product of the abscissæ of the first axis and the other axis.

198.—Lateral frustum or solid zone of prolate spheroid by planes parallel to each other, and to longer or fixed axis.

Coved ceiling of elliptical plan, etc.; reversed: a flat-bottomed boat, a scow, a dish, basket, etc., etc.

Its parallel bases and middle section, **similar ellipses**; for areas of which see "Key to Ster.," page 51. Its lateral area resolvable into **continuous trapeziums** of varying height if parallel to bases, but of uniform height, if lines be drawn from extremities of fixed axis.

199.—Segment of prolate spheroid by a plane inclined to axis.

Liquid in spheroidal vessel inclined from the vertical, a scoop, scuttle, etc.

Its base and middle section, similar ellipses; its other base, a point; its spheroidal surface resolvable by circles drawn from extremity of fixed axis into a circle, trapeziums and triangle.

200.—Frustum of prolate spheroid between non-parallel planes.

The one, perpendicular to fixed axis, the other oblique or inclined thereto.

Decompose into frustum with parallel bases, and an ungula. Compute separately, and add; or compute whole segment due to frustum and deduct lesser segment.

The areas of Spherical Triangles and Polygons to any radius or diameter.

The following will be found a new, easy and concise rule

By MR. C. BAILLARGE,

for finding the area of any spherical triangle, or of a triangle described on the surface of a sphere of any given diameter.

The area of a sphere to diameter 1. being	= 3.141,592,653,589,793+
Dividing by 2, we get that of the hemisphere	= 1.570,796,326,794,896,5
This divided by 4 = area of tri-rectgl'r sph. triangle	= 0.392,699,081,698,724,1
÷ 90 = area of 1° or of bi-rect. sph. tri. with sp. ex. = 1°	= 0.004,363,323,129,985,8
÷ 60 = " of 1' or of " " " " 1'	= 0.000,072,722,052,166,43
÷ 60 = " of 1" or of " " " " 1"	= 0.000,001,212,034,202,77
÷ 10 = " of 0.1" or of " " " " 0.1"	= 0.000,000,121,203,420,277
÷ 10 = " of 0.01" or of " " " " 0.01"	= 0.000,000,012,120,342,027,7
÷ 10 = " of 0.001" or of " " " " 0.001"	= 0.000,000,001,212,034,202,77

Find the spherical excess, that is, the excess of the sum of the three spherical angles over two right angles, or from the sum of the three spherical angles deduct 180°. Multiply the remainder, that is, the spherical excess, by the tabular number herein above given: the degrees by the number set opposite to 1°, the minutes by that corresponding to 1' and so on of the seconds and fractions of a second; add these areas and multiply their sum by the square of the diameter of the sphere of the surface of which the given triangle forms part, the result is the area required.

EXAMPLE.

Let the spherical excess of a triangle described on the surface of a sphere of which the diameter is an inch, a foot, or a mile, etc., be 3° — 4' — 2.235". What is the area?

Area of 1° = 0.004,363,323,129,985,8	×	3° = 0.013,089,969,389,955
" 1' = 0.000,072,722,052,166,43	×	4' = 0.000,290,838,208,664
" 1" = 0.000,001,212,034,202,	×	2" = 0.000,002,424,068,404
" 0.1" = 0.000,000,121,203,420,	×	0.2" = 0.000,000,242,406,840
" 0.01" = 0.000,000,012,120,342,	×	0.03" = 0.000,000,036,361,026
" 0.001" = 0.000,000,001,212,034,	×	0.005" = 0.000,000,006,060,170
Area required		0.013,383,566,495,059

The answer is of course in square units or fractions of a square unit of the same name with the diameter. That is, if the diameter is an inch, the area is the fraction of a square inch ; if a mile, the fraction of a square mile, and so on.

Remark.—If the decimals of seconds are neglected, then of course the operation is simplified by the omission of the three last lines for tenths, hundredths and thousandths of a second or of so many of them as may be omitted.

If the seconds are omitted, as would be the case in dealing with any other triangle but one on the earth's surface, on account of its size ; there will in such case remain only the two upper lines for degrees and minutes, which will prove of ample accuracy when dealing with any triangular space, compartment, or component section of a sphere of the size of a dome, vaulted ceiling, gasometer, or large copper or boiler, etc. ; and in dealing with such spheres as a billiard or other playing ball, a cannon ball or shell, the ball of a vane or steeple, or any boiler, copper, etc., of ordinary size, it will generally suffice to compute for degrees only. Whence the following.

RULE to degrees only

Multiply the spherical excess in degrees by 0.004,363 and the result by the square of the diameter for the required area. For greater accuracy case—0.004,363,323.

RULE to degrees and minutes.

Proceed as by last rule for degrees. Multiply the spherical excess in minutes by 0.000,073, or for greater accuracy by 0.000,072,722. Add the results, and multiply their sum by the square of the diameter for the required area.

EXAMPLE I.

Sum of angles $140^\circ + 92^\circ + 68^\circ = 300$; $300 - 180 = 120^\circ$ spherical excess, diameter = 30, page 124 of "Key to Ster." Answer area of 1° 0.004,363
Multiply by spherical excess 120°

	We get	0.523,560
This multiplied by square of diameter 30	=	900
Required area	=	471.194,000

A result correct to units and agreeing therewith with the answer given in the "Key," which is 471.24. If now greater accuracy be required, it is to be obtained by taking in more decimals, thus, say area 1° = 0.004,363,323

120
0.523,598,760
900
471.238,884,000

EXAMPLE II.

"Key to Ster.," page 124. The three angles each 120° , their sum 360° , from which deducting 180° we get spherical excess $= 180^\circ$. Diameter 20, of which the square $= 400$.

Answer.....	Area to $1^\circ = 0.004,363,323$
	180
	<hr/> 0.785,398,140
	400

Answer in "Key," 314.16

314.159,256,000

EXAMPLE III.

See example 4, page 125, "Key to Ster." The sum of the three angles of a triangle traced on the surface of the Terrestrial sphere exceeds by ($1''$) one second, 180° ; what is the area of the triangle, supposing the earth to be a perfect sphere with a diameter $= 7,912$ English miles, or, which is the same thing, that the diameter Terrestrial spheroid or of its osculatory circle at the given point on its surface be 7,912 miles.

Answer.	Area of $1''$ to diameter 1. $=$	0.000,001,212,034,202
	Square of diameter	<hr/> 62,598,744
		75.871,818,730,242,288

The answer in the "Key," is 75.87321 square miles, which is only correct to the second place of decimals from having assumed $\pi = 3.1416$, instead of, as it should be for greater accuracy, 3.141,592,653,589,793, etc.

Remark.—This unit 75.87 etc., as applied to the Terrestrial sphere, as explained page 125, par. 166 of "Key to Ster.," becomes a tabular number, which may be used for computing the area of any triangle on the earth's surface, as it evidently suffices to multiply the area 75.87 etc., corresponding to one second ($1''$) by the number of seconds in the spherical excess to arrive at the result; and as also stated in "Key," the result may be had true to the tenth, thousandth, or millionth of a second, or of any other fraction thereof, by successively adding the same figures 75.78 etc., with the decimal point shifted to the left one place for every place of decimals in the given fraction of such second: the tenth of a second giving 7.578 etc., square miles, the $0.01'' = .7578$ of a square mile, the $0.001'' = .07578$ etc., of a square mile, and so on; while, by shifting the decimal point to the right, we get successively $10'' = 757.8$ square miles, $100'' = 7578$ etc., square miles, or $1' = 75.78 \times 60''$, $1^\circ = 75.78$, etc., $\times 60'' \times 60''$.

RULE.

To compute the area of any spherical polygon.

Divide the polygon into triangles, compute each triangle separately by the foregoing rules for triangles and add the results.

Or,

From the sum of all the interior angles of the polygon subtract as many times two right angles as there are sides less two. This will give the spherical excess. This into the tabular area for degrees, minutes, seconds and fractions of a second, as the case may be, and the sum of such areas into the square of the diameter of the

sphere on which the polygon is traced will give the correct area of the proposed figure.

For example see page 128 of "Key to Ster."

It may be remarked here that the area of a spherical lune or the convex surface of a spherical ungula is equal to the tabular number into twice the spherical excess since it is evident that every such lune is equivalent to two bi-rectangular spherical triangles of which the angle at the apex, that is the inclination of the planes forming the ungula, is the spherical excess.

REMARK.—The area found for any given spherical excess, on a sphere of given diameter, may be reduced to that, for the same spherical excess, on a sphere of any other diameter; these areas being as the squares of the respective diameters.

The area found for any given spherical excess on the earth's surface, where the diameter of the osculatory circle is supposed to be 7912 miles, may be reduced to that for the same spherical excess where the osculatory circle is of different radius; these areas being as the squares of the respective radii or diameters.

N.B. The "Key to Ster." also sets forth how the converse operation is performed of obtaining the spherical excess from the area.

